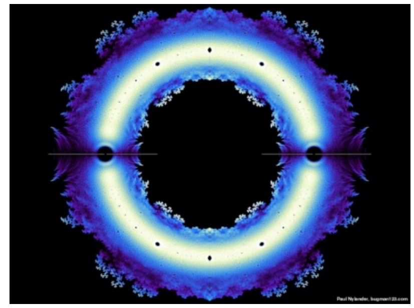
5.8 Fundamental Theorem of Algebra

Practice Tasks



I. Concepts and Procedures

1. Find the two square roots of each complex number by creating and solving polynomial equations.

a) $z = 15 - 8i$

b) $z = 8 - 6i$

c) $z = -3 + 4i$

d) $z = -5 - 12i$

e) $z = 21 - 20i$

f) $z = 16 - 30i$

g) $z = i$

II. Problem Solving

A *Pythagorean triple* is a set of three positive integers a, b, and c such that $a^2 + b^2 = c^2$. Thus, these integers can be the lengths of the sides of a right triangle.

1. Show algebraically that for positive integers p and q, if

$$a = p^2 - q^2$$
$$b = 2pq$$
$$c = p^2 + q^2$$

then $a^2 + b^2 = c^2$

2. Select two integers p and q, use the formulas in Problem 8 to find a, b, and c, and then show those numbers satisfy the equation $a^2 + b^2 = c^2$.

3. Use the formulas from Problem 8, and find values for p and q that give the following famous triples.

   a. (3,4,5)

   b. (5,12,13)

   c. (7,24,25)

   d. (9,40,41)

4. Is it possible to write the Pythagorean triple (6,8,10) in the form $a = p^2 - q^2$, $b = 2pq$, $c = p^2 + q^2$ for some integers p and q? Verify your answer.

5. Choose your favorite Pythagorean triple (a, b, c) that has a and b sharing only 1 as a common factor, for example (3,4,5), (5,12,13), or (7,24,25),… Find the square of the length of a square root of $a + bi$; that is, find $|p + qi|^2$, where $p + qi$ is a square root of $a + bi$. What do you observe?

III. Modeling

1. Write a function of 4th degree with an imaginary zero and an irrational zero.